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MATDIP301

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019

Advanced Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. If $Z = \frac{(2+3i)(1-i)}{(1+i)}$, find $z+\bar{z}$ and $z\bar{z}$. **(06 Marks)**

b. Express the complex number $1+\sqrt{3}i$ in the polar form and exponential form. **(07 Marks)**

c. Find the modulus and amplitude of $\frac{(1+2i)(2-i)}{4+3i}$. **(07 Marks)**

2 a. Find the n^{th} derivative of $y = \frac{x+2}{x+1} + \log\left(\frac{x+2}{x+1}\right) + \cos^2 2x$. **(06 Marks)**

b. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

c. Find the angle between the curves $\gamma = a(1+\cos\theta)$ and $\gamma = a(1-\cos\theta)$. **(07 Marks)**

3 a. Obtain the Maclaurin's series for $\sqrt{1+\sin 2x}$ upto the term involving x^4. **(06 Marks)**

b. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. **(07 Marks)**

c. If $u = x+3y^2-z^3$, $v = 4x^2yz$, $w = 2z^2-xy$, evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, –1, 0). **(07 Marks)**

4 a. If ϕ is the angle between the radius vector and tangent to the polar curve r = t(θ) at (r, θ), show that $\tan\phi = r\frac{d\theta}{dr}$. **(06 Marks)**

b. If v = f(x – y, y – z, z – x), show that $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = 0$. **(07 Marks)**

c. State and prove Euler's theorem for a homogeneous function u in x and y of degree n. **(07 Marks)**

5 a. Obtain the reduction formula for $\int \sin^n x\,dx$ and hence evaluate $\int \sin^3 x\,dx$. **(06 Marks)**

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax-x^2}\,dx$. **(07 Marks)**

c. Evaluate $\int_0^1\int_x^{\sqrt{x}} (x^2+y^2)\,dy\,dx$. **(07 Marks)**

6 a. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)dydxdz$. **(06 Marks)**

b. Show that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

c. Evaluate $\int_{0}^{\pi/2}\sqrt{\tan\theta}\,d\theta$. **(07 Marks)**

7 a. Solve $e^x \tan y dx+(1+e^x)\sec^2 y\, dy=0$. **(06 Marks)**

b. Solve $x\frac{dy}{dx}+y=x^3y^6$. **(07 Marks)**

c. Solve $(3x^2+6xy^2)dx+(6x^2y+4y^3)dy=0$. **(07 Marks)**

8 a. Solve $\frac{d^3y}{dx^3}+4\frac{dy}{dx}=\sin 2x$. **(06 Marks)**

b. Solve $(D-2)^2y=8(e^{2x}+x^2)$. **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=e^{-2x}\cos 2x$ **(07 Marks)**

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